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The *JMETC* is a re-creation of an earlier publication by the Teachers College Columbia University Program in Mathematics. As a peer-reviewed, semi-annual journal, it is intended to provide dissemination opportunities for writers of practice-based or research contributions to the general field of mathematics education. Each issue of the *JMETC* will focus upon an educational theme. The themes planned for the 2012 Fall-Winter and 2013 Spring-Summer issues are *Equity* and *Leadership*, respectively.

*JMETC* readers are educators from pre-K-12 through college and university levels, and from many different disciplines and job positions—teachers, principals, superintendents, professors of education, and other leaders in education. Articles to appear in the *JMETC* include research reports, commentaries on practice, historical analyses, and responses to issues and recommendations of professional interest.

**Manuscript Submission**  
*JMETC* seeks conversational manuscripts (2,500-3,500 words in length) that are insightful and helpful to mathematics educators. Articles should contain fresh information, possibly research-based, that gives practical guidance readers can use to improve practice. Examples from classroom experience are encouraged. Articles must not have been accepted for publication elsewhere. To keep the submission and review process as efficient as possible, all manuscripts may be submitted electronically at www.tc.edu/jmetc.

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Call for Papers
The “theme” of the fall issue of the Journal of Mathematics Education at Teachers College will be Equity. This “call for papers” is an invitation to mathematics education professionals, especially Teachers College students, alumni and friends, to submit articles of approximately 2500-3500 words describing research, experiments, projects, innovations, or practices related to equity in mathematics education. Articles should be submitted to Ms. Krystle Hecker at JMETC@tc.columbia.edu by September 1, 2012. The fall issue’s guest editor, Mr. Nathan N. Alexander, will send contributed articles to editorial panels for “blind review.” Reviews will be completed by October 1, 2012, and final manuscripts of selected papers are to be submitted by October 15, 2012. Publication is expected by November 15, 2012.

Call for Volunteers
This Call for Volunteers is an invitation to mathematics educators with experience in reading/writing professional papers to join the editorial/review panels for the fall 2012 and subsequent issues of JMETC. Reviewers are expected to complete assigned reviews no later than 3 weeks from receipt of the manuscripts in order to expedite the publication process. Reviewers are responsible for editorial suggestions, fact and citations review, and identification of similar works that may be helpful to contributors whose submissions seem appropriate for publication. Neither authors’ nor reviewers’ names and affiliations will be shared; however, editors'/reviewers’ comments may be sent to contributors of manuscripts to guide further submissions without identifying the editor/reviewer.

If you wish to be considered for review assignments, please request a Reviewer Information Form. Return the completed form to Ms. Krystle Hecker at hecker@tc.edu or Teachers College Columbia University, 525 W 120th St., Box 210, New York, NY 10027.

Looking Ahead
Anticipated themes for future issues are:

Fall 2012    Equity
Spring 2013    Leadership
Fall 2013    Modeling
Spring 2014    Teaching Aids

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Assessment of Mathematical Modeling

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Modeling is now being given prominence as one of the six high school subject areas in the Standards for Mathematical Content in the Common Core State Standards for Mathematics (CCSSM). The CCSSM document suggests that mathematical modeling should not be taught as an isolated topic but instead should be employed along with other topics. The essence of a modeling problem is constructing a mathematical model for the situation at hand. Assessing mathematical modeling is a new field that requires more study. Developing more tools and ideas to assess modeling tasks is essential and beneficial for mathematics teachers. Rubrics that assess the modeling process are a good beginning in finding suitable methods of assessing mathematical modeling. This article provides two methods for assessing modeling tasks. The first is a scoring rubric that is based on the process of CCSSM’s modeling cycle while the second instrument intends to assess the affective domain of mathematical modeling.

Keywords: assessment, mathematical modeling, rubrics, CCSSM, modeling tasks.

Introduction

Mathematical problem solving and mathematical modeling have come a long way since Polya’s 1945 description of the problem solving heuristics in How to Solve It. Problem solving in mathematics and modeling in mathematics are two related but different areas within mathematics education.

Mathematics educators around the world have placed problem solving as the focus of mathematics curricula for some time, especially since the 1980s. In the United States, the National Council of Teachers of Mathematics (NCTM) produced the Curriculum and Evaluation Standards for School Mathematics Evaluation Standards (1989) and problem solving was one of the main process standards. The push for problem solving became even stronger in 2000 with the document Principles and Standards in School Mathematics that asserted that “problem solving should be the central focus of the mathematics curriculum” (NCTM, 2000, p. 52).

Where does mathematical modeling fit in? Modeling was highlighted in the Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989). Educational organizations such as the Consortium for Mathematics and Its Applications (COMAP), the University of Chicago School Mathematics Project (UCSMP), and the Systemic Initiative for Montana Mathematics and Science (SIMMS) have produced educational books and lessons on mathematical modeling (Hodgson, 1995). Recently, the strongest support for modeling was the development of the national standards for K–12 mathematics known as the Common Core State Standards for Mathematics (CCSSM) in 2010. Modeling is now being given prominence as one of the six high school subject areas in the Standards for Mathematical Content in the CCSSM. Furthermore, modeling with mathematics is also included as one of the Standards for Mathematical Practice which “describe varieties of expertise that mathematics educators at all levels should seek to develop in their students” (NGACBP & CCSSO, 2010, p. 6).

If mathematical modeling is so prominent in curricula, why isn’t it implemented in the mathematics classroom more often? One reason is that mathematical modeling is a challenging task that requires various competencies, including problem solving. There might be general steps in solving a modeling problem but no direct algorithm. Mathematical modeling requires real-world knowledge from domains that may not be as familiar to students and teachers, making solutions less predictable and linear (Burkhardt, 2004; DeLange, 1987; Ikeda, 2007; Pollak, 1979). Furthermore, modeling tasks are not like the typical “word problems” we have in textbooks. Though they are useful, word problems often only serve as an exercise (Blum, 2011).

Another challenge mathematics teachers face is modeling assessment. Anecdotal evidence indicates that most of the time, mathematics lessons focus on a single, final product: the correct solution. Problem solving tasks, for instance, should result in an easy-to-describe solution. Mathematical modeling, however, does not always yield a simple, precise answer, or, the correct solution. Because of this, the question among many mathematics educators is “how do we assess mathematical modeling and, in particular, the mathematical modeling process?” Little has been written on the assessment of mathematical modeling and the purpose of this article is to provide some ideas on its assessment.
Assessing Competency in Mathematical Modeling

PISA studies (OECD, 2005, 2007) have indicated that students struggle and have difficulty with modeling tasks around the world. Based on previous studies, mathematical modeling competency is defined as “the ability to identify relevant questions, variables, relations or assumptions in a given real-world situation, to translate these into mathematics and to interpret and validate the solution of the resulting mathematical problem in relation to the given situation” (Blum, Galbraith, Henn, & Niss, 2007, p. 12).

Blum (2011) discusses examples of students’ difficulties in modeling tasks based on the six steps of the modeling cycle in the DISUM studies. He asserted that “it is a particularly challenging open research question to establish a theoretically and empirically based competence model for mathematics modeling” (p. 21). PISA began by defining the six proficiency levels in mathematics for assessing the mathematical literacy of students in various countries. The assessment also included modeling as one of the eight characteristic mathematical competencies besides mathematical argumentation, mathematical thinking and reasoning, problem posing and solving, representation, communication, symbol and formalism, and aids and tools that are relevant and meaningful across all educational sectors (OECD, 2005). In Germany, the modeling proficiencies are listed in five levels, namely, applying simple standard models, direct modeling from familiar contexts, few-step modeling, multi-step modeling, and complex modeling (Blum, 2011).

The Mathematical Modeling Process

The essence of a modeling problem is constructing a mathematical model of a real-world situation at hand, understanding it mathematically, then interpreting those results in the context of the original situation. This might not be as simple as a typical problem solving task. Furthermore, problems in a real-world context require adequate mathematical and extra-mathematical knowledge. With an explicit modeling process, students are able to understand the process and develop strategies while attempting to solve the problem.

Description of the DISUM Modeling Cycle

As Blum and Leiss (2008) state, the purpose of the Didaktische Interventionsformen für einen selbständigkeitsorientierten Unterricht in Mathematik (“didactical intervention modes for mathematics teaching oriented towards self-regulation”) project (DISUM) is to “[i]nvestigate how students and teachers deal with cognitively demanding modeling tasks and [determine] what effects various learning environments for modeling have on students’ competency development” (p. 17). The essential features of DISUM’s description of the modeling process are shown in Figure 1.

Description of the CCSSM Modeling Cycle

In the CCSSM, a model can be something simple to describe a product or relationship between variables. The modeling cycle begins with

1. identifying variables in the situation and selecting those that represent essential features,
2. formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables,
3. analyzing and performing operations of these relationships to draw conclusions,
4. interpreting the results of the mathematics in terms of the original solution,
5. validating the conclusion by comparing them with the situation, and then either improving the model or, if is acceptable, (6) reporting on the conclusions and the reasoning behind them.

Choices, assumptions and approximations are present throughout this cycle (NGACBP & CCSSO, 2010, p. 72; see Figure 2).

How to Assess Mathematical Modeling Tasks

Assessment in mathematics education generally refers to the “judging of the mathematical capability, performance and achievement of students” (Niss, 1992, p. 3). Given the complexity of a mathematical modeling task, however, how does one assess it?
ASSESSMENT NOTES FROM THE FIELD

Several studies on assessing mathematical modeling have focused on developing scales that assess the model creation process, assessing specific mathematical ideas used in a model, and developing a consistent instrument for different problems (Keck, 1996). Often, educators focus on the products of the modeling process, but this is not sufficient, as the process of mathematical modeling is also very important. For a modeling task, the process can be broken down into two parts, first the general idea followed by the specific ones.

Two suggestions for assessing modeling tasks are presented here. The first is a scoring rubric that is based on the process of CCSSM’s modeling cycle. The second instrument intends to assess the affective domain of mathematical modeling.

A Scoring Rubric Using CCSSM’s Modeling Cycle

To address students’ competency in the process of mathematical modeling, the author suggests a rubric to assess the modeling tasks. The suggested outline of the rubric informs the educator of the importance of looking for the steps while solving the modeling tasks. Assessing the steps of the process of solving the modeling task provides a picture of how students think to solve it. Even though the final solution is not achieved in a high percentage of “correct” answers—if such a thing exists for the given problem—as is usual in typical examinations, nonetheless this rubric might provide some new insight of the process of solving modeling tasks.

The rubric (Table 1. Modeling Cycle Scoring Rubric) was developed after considering the important checklist that is required in each process of the CCSSM modeling cycle. Certain checklist items were adapted from Keck’s (1987) scoring scale of mathematical modeling projects. For each process, a 5-point rating score is used. The possible rating score is from 0 to 4 and assigned as follows.

- 0: Not done
- 1: Below acceptable
- 2: Average
- 3: Good
- 4: Excellent

Each process is weighted and based on the importance of the process assessed in the modeling cycle. Each item within every process is given 0–4 points. For example, in the “Identifying Variables” section, the greatest possible score is 12 points since each of the three items gets a 0 to 4 score with a weight of 1. The author suggests that heavier weights should be given for formulating a model and interpreting the results since these are extremely important steps in the modeling process.

Assessing the Affective Domain of Mathematical Modeling

A novel method educators can use to assess the modeling process is assessing the student’s appreciation of the subject. This assessment is very different from the usual test items found on state examinations or the standardized tests and it needs to be investigated and developed further. It is adapted from the field of art appreciation. When we look at the field, it is not so much about the final product but how we admire and appreciate the art work. This can be applied in the modeling process. The following are some of the questions researchers should ask regarding appreciation of mathematical modeling. These questions were adapted from Faulkner’s (1939) work on art appreciation.

- What is mathematical modeling appreciation?
- What are the specific objectives of mathematical modeling tasks in terms of student behavior?
- How can various stages of the development of mathematical modeling appreciation be measured or observed?
- How can mathematical modeling appreciation be integrated with other subjects?

In assessing modeling tasks, an affective test similar to the Fenema-Sherman (1976) Mathematics Attitude Scale can be created to measure attitudes towards mathematical modeling. One simple method is to give 12
items on mathematical modeling appreciation using the 5-point Likert scale (1932) after students complete modeling tasks. This can be a simple way of assessing students’ attitudes on the modeling problem. The instrument for testing mathematical modeling attitudes can be divided into four domains such as Enjoyment, Self-Confidence, Value, and Motivation (Tapia, 2004; see Table 2. Affective Test Domains and Items). Each domain contains several items and students can rate the items with a 5-point Likert scale from Strongly Agree to Strongly Disagree.

A potential flaw in assessing students with an attitude test is that they may just indicate all positive answers to appease the teacher. Thus, it might not be the true reflection of what the student thinks about mathematical modeling. Giving this attitude test after each modeling task has been solved might help overcome this problem of appeasing teachers. The attitude test can be developed further as an alternative assessment for mathematical modeling.

Conclusion

Assessing mathematical modeling is a new field that requires more study with the implementation of CCSSM. Developing more tools and ideas to assess modeling tasks is essential and beneficial for mathematics teachers. Rubrics that assess the modeling process constitute a good beginning in finding appropriate methods of assessing mathematical modeling. In addition, attitude tests that describe a student’s appreciation of mathematical modeling are also suggested. It is hoped that these ideas can be tested and developed further.

References


Table 1. Modeling Cycle Scoring Rubric

<table>
<thead>
<tr>
<th>Process</th>
<th>Score</th>
<th>Weight</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Identifying Variables</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. States the variables in the model</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. States problem clearly</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. States important features</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Formulating a Model</td>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. Creates a model</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. Clearly states all assumptions</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Describes relationships between variables</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mathematical Operations</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. Correct use of mathematics</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. Analyzes relationships between variables</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Performs operations on the variables and relationships</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Interpreting the Results</td>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. Reaches solution</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. Interprets solution</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Evaluates model and solution</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Validating the Conclusion</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. Revises the model based on the problem</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. Interprets solution based on the revised model</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Improves the model</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reporting on Conclusions</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. Summarizes the results</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. Reasons about assumptions</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>


### Table 2. Affective Test Domains and Items

<table>
<thead>
<tr>
<th>Domain</th>
<th>Items</th>
</tr>
</thead>
<tbody>
<tr>
<td>Enjoyment</td>
<td>• I enjoy solving modeling tasks.</td>
</tr>
<tr>
<td></td>
<td>• I can see how mathematics works with modeling tasks.</td>
</tr>
<tr>
<td></td>
<td>• Modeling tasks are difficult and boring.</td>
</tr>
<tr>
<td>Self-Confidence</td>
<td>• Studying mathematical modeling makes me feel nervous.</td>
</tr>
<tr>
<td></td>
<td>• I am always under a terrible stress in a mathematics modeling lesson.</td>
</tr>
<tr>
<td></td>
<td>• I am able to solve modeling tasks without too much difficulty.</td>
</tr>
<tr>
<td>Value</td>
<td>• Mathematical modeling problems are real-world problems.</td>
</tr>
<tr>
<td></td>
<td>• Mathematical modeling is one of the most important subjects for people to study.</td>
</tr>
<tr>
<td></td>
<td>• Mathematical modeling courses would be very helpful no matter what I decide to study.</td>
</tr>
<tr>
<td></td>
<td>• I appreciate mathematics more after solving the modeling tasks.</td>
</tr>
<tr>
<td>Motivation</td>
<td>• I applied my mathematical knowledge in modeling problems.</td>
</tr>
<tr>
<td></td>
<td>• I can see how mathematics is used in solving real-world problems.</td>
</tr>
<tr>
<td></td>
<td>• I am willing to take more mathematics courses than required.</td>
</tr>
</tbody>
</table>


